Linear Algebra-Determinants

The determinant of a matrix is a scalar value that can be computed from the elements of the matrix. It's a fundamental concept in linear algebra, and it has numerous applications in various fields such as physics, engineering, economics, and computer science.

What does a determinant represent?

In essence, the determinant of a square matrix (i.e., a matrix with an equal number of rows and columns) represents the scaling factor by which the matrix transforms volumes. In other words, it's a measure of how much the linear transformation represented by the matrix expands or shrinks the space.

Properties of Determinants

Determinants have several important properties:

1. **Existence**: A square matrix always has a determinant.
2. **Uniqueness**: The determinant is unique for each matrix.
3. **Non-negativity**: The determinant is non-negative (0 or positive) if and only if the matrix is invertible.
4. **Multiplicativity**: The determinant of a product of matrices is equal to the product of their determinants.

How to compute Determinants?

There are several methods to compute determinants:

1. **Cofactor Expansion**: Expand along rows or columns, using cofactors and minors.
2. **Rule of Sarrus**: Apply this rule for 3x3 matrices only.
3. **LU Decomposition**: Use the LU decomposition algorithm.

Let's consider an example:

Example:

Find the determinant of the matrix:

A = | 1 2 | | -3 4 |

To compute the determinant, we'll use cofactor expansion along the first row:

det(A) = (1)(-3)^0 × (-3)^1 + (2)(-3)^0 × (4)^1 = -(3) + (8) = 5

Thus, the determinant of matrix A is 5.

Note that this example illustrates a simple case. For larger matrices, the cofactor expansion method can become quite cumbersome.

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