Linear Algebra-Eigenvalues and Eigenvectors

In linear algebra, **eigenvalues** and **eigenvectors** are two fundamental concepts that help us understand the behavior of linear transformations.

Eigenvectors:

An **eigenvector** is a non-zero vector that, when multiplied by a square matrix (linear transformation), results in a scaled version of itself. In other words, if `A` is a square matrix and `v` is an eigenvector, then:

`Av = λv`

where `λ` (lambda) is the **eigenvalue** associated with the eigenvector `v`.

Eigenvalues:

An **eigenvalue** is a scalar value that represents how much a linear transformation changes the direction and magnitude of an input vector. In other words, eigenvalues tell us how much each eigenvector is scaled by the linear transformation.

Example:

Suppose we have a matrix `A`:

`| 2 1 | | 4 -3 |`

Let's find the eigenvalues and eigenvectors of this matrix.

To do this, we can use the characteristic equation:

`det(A - λI) = 0`

where `I` is the identity matrix.

Solving for `λ`, we get:

`(-λ + 2)(-3 - 3λ) = 0`

This gives us two possible eigenvalues: `-3/1 = -3` and `2/2 = 1`.

Now, let's find the corresponding eigenvectors:

For `λ = -3`, we need to solve:

`(A + 3I)v = 0`

Substituting the matrix `A` and solving for `v`, we get:

`v = | -1 | | 1 |`

So, one eigenvector associated with `λ = -3` is `v = [-1, 1]`.

Similarly, for `λ = 1`, we need to solve:

`(A - I)v = 0`

Substituting the matrix `A` and solving for `v`, we get:

`v = | 2 | | 4 |`

So, one eigenvector associated with `λ = 1` is `v = [2, 4]`.

Conclusion:

In this example, we found two eigenvalues (`-3` and `1`) and their corresponding eigenvectors (`[-1, 1]` and `[2, 4]`). These values help us understand the behavior of the linear transformation represented by matrix `A`.

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