Calculus-Integration by Substitution

Integration by substitution is a technique used to evaluate definite integrals by changing the variable of integration. This method allows us to simplify complex integrals and make them more manageable.

The Basic Idea

Suppose we have an integral of the form:

$\int f(x)\,dx$

We want to substitute $u = g(x)$, where $g(x)$ is a function that simplifies $f(x)$. Then, we find du/dx and rewrite the original integral in terms of u.

The Formula

If $u = g(x)$, then:

$du/dx = g'(x)$

$dx = 1 / (du/dx)$

Substituting these expressions into the original integral, we get:

$\int f(u) * (du/dx)^{-1}\,du$

This is equivalent to:

$\int f(u) * 1/(g'(x))\,du$

Example:

Evaluate the integral $\int (2x+3)^5\,dx$ from $x=0$ to $x=2$.

Let's use substitution with $u = 2x + 3$. Then, $du/dx = 2$, and:

$dx = 1 / (du/dx) = 1/2$

We can now rewrite the integral in terms of u:

$\int (u)^5 * (1/2)\,du$

This becomes a much simpler integral to evaluate:

$(1/2) \int u^5\,du$

The antiderivative is:

$(1/6)u^6 + C$

Now, we need to substitute back in terms of x. We have $u = 2x + 3$, so:

(1/6)(2x+3)^6 from 0 to 2

Substituting the limits of integration, we get:

[(1/6)(35)] - [(1/6)(27)]

Simplifying this expression gives us the final answer:

5.17 (rounded to two decimal places)

This example demonstrates how integration by substitution can be used to simplify complex integrals and evaluate them more easily.

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