Linear Algebra-Orthogonality and Orthonormality

On the other hand, a set of vectors is said to be **orthonormal** if they are both orthogonal to each other and have a norm (or length) of 1.

Mathematically:

- Two vectors `a` and `b` are orthogonal if their dot product is zero: `a · b = 0`
- A set of vectors `{v1, v2, ..., vn}` is orthonormal if:
  - Each vector `vi` is orthogonal to every other vector `vj (i ≠ j)`: `vi · vj = 0` for all `i ≠ j`
  - Each vector `vi` has a norm of 1: ||vi|| = 1 for all `i`

Example:

Suppose we have two vectors in $\mathbb{R}^2$, represented as column matrices:

a = [1, 2] b = [-2, 1]

To check if they are orthogonal, we compute their dot product:

a · b = (1)(-2) + (2)(1) = -2 + 2 = 0

Since the dot product is zero, vectors `a` and `b` are orthogonal.

Now, let's consider a set of three vectors in $\mathbb{R}^3$:

v1 = [1, 0, 0] v2 = [0, 1, 0] v3 = [0, 0, 1]

To check if they form an orthonormal set, we need to verify two conditions:

1. Orthogonality: Each pair of vectors must be orthogonal.

v1 · v2 = (1)(0) + (0)(1) + (0)(0) = 0 v1 · v3 = (1)(0) + (0)(0) + (0)(1) = 0 v2 · v3 = (0)(0) + (1)(0) + (0)(1) = 0

All pairs of vectors are orthogonal, so they satisfy the first condition.

2. Norm-1: Each vector must have a norm of 1.

||v1|| = sqrt((1)^2 + (0)^2 + (0)^2) = sqrt(1) = 1 ||v2|| = sqrt((0)^2 + (1)^2 + (0)^2) = sqrt(1) = 1 ||v3|| = sqrt((0)^2 + (0)^2 + (1)^2) = sqrt(1) = 1

All vectors have a norm of 1, so they satisfy the second condition.

Since both conditions are satisfied, the set of vectors `{v1, v2, v3}` is orthonormal.